Exam. Code : 211001

Subject Code : 3838

M.Sc. Mathematics I[st] Semester

DIFFERENTIAL EQUATIONS

Paper : MATH-555

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **five** questions consisting of **one** from each section and **fifth** question can be attempted from any section.

SECTION—A

1. (i) Find the characteristic values and characteristic functions of the Strum-Liouville Problem

$$\frac{d^2y}{dx^2}+\lambda y=0,\ y(0)-y'(0)=0,\ y(\pi)-y'(\pi)=0.$$ 10

(ii) State and prove Sturm's Fundamental Comparison Theorem. 10

2. (i) Check the existence and uniqueness of solution of the initial value problem $\frac{dy}{dx}=y^2, y(1)=-1.$ Hence solve it. 10

(ii) (a) Find the orthogonal trajectories of the family of parabolas $y = cx^2$ and draw its diagram. 5

(b) Compute the first four successive approximations of the problem $y' - 1 + xy$, $y(0) - 1$. 5

SECTION—B

3. (i) State and prove Convolution Theorem for Laplace Transform. 10

(ii) Solve the initial-value problem using Laplace transform $\frac{dY}{dt} - 2Y = e^{5t}, Y(0) = 3$. 10

4. (i) Let F be a real valued function which is continuous for $t \geq 0$ and of exponential order c^{at}. Let F' be piecewise continuous in every finite closed interval $0 \leq t \leq b$. Then L(F') exists for $s > a$ and $L\{F'(t)\} = sL\{F(t)\} - F(0)$. 10

(ii) (a) Find $L^{-1}\left[\frac{1}{s^2 \mid \cos \mid 13}\right]$. 5

(b) Find $L\{\sin at \sin bt\}$. 5

SECTION—C

5. (i) (a) Define Fourier transform and its inverse transform. 2

(b) State and prove the linear property of Fourier transform. 4

(c) State and prove the shifting property of Fourier transform. 4

(ii) State and prove the convolution theorem for Fourier Transforms ; also find the Fourier Cosine transform of e^{-ax}, $a > 0$. 10

6. (i) Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & |x|<1 \\ 0, & |x|>1 \end{cases}$. Hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$. 10

(ii) Solve the differential equation $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 2y = \cos t$ using Fourier transform. 10

SECTION—D

7. (i) Find the generating function for Hermite polynomial and establish the relation between $H_n(x)$ and $H_n(-x)$ using generating functions. 10

(ii) State and prove Rodrigue's Formula for Legendre Polynomials. 10

8. (i) Obtain the orthogonality relating for Laguerre polynomials. 10

(ii) Prove that $e^{\frac{x}{2}\left(u-\frac{1}{u}\right)}$ is generating function of Bessel functions. 10